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OBSERVER DESIGN FOR A CLASS OF MIMO NONLINEAR SYSTEMS (PREPRINT)

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14. ABSTRACT
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Observer Design for a Class of MIMO Nonlinear Systems

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***Abstract*— Under the boundedness and observability conditions, we present a globally convergent observer for a class of multi-output nonlinear systems which covers the block-triangular observer forms studied previously in the literature. The result presented in this paper incorporates and generalizes the earlier work on the observer design for single-output observable systems. Extensions to detectable systems and controlled systems are also considered. Examples are given to illustrated the validity of proposed method.**

***Index Terms*— Nonlinear systems, dynamic high-gain observers, universal control, observability and detectability, boundedness.**

I. Introduction

In this paper, we are interested in the problem of global observer design for a multi-output nonlinear system in the observable canonical form

$$\begin{aligned}\dot{x}_{i1} &= x_{i2}\\ \dot{x}_{i2} &= x_{i3}\\ &\vdots\\ \dot{x}_{i,k_i-1} &= x_{i,k_i}\\ \dot{x}_{i,k_i} &= f_i(x), \qquad i=1,2,\cdots,p\\ y &= \left(\ y_1,y_2,\cdots,y_p\ \right)^T = \left(\ x_{11},x_{21},\cdots,x_{p1}\ \right)^T\end{aligned} \tag{1.1}$$

where $x=(x_1,x_2,\cdots,x_p)^T,\ x_i=(x_{i1},x_{i2},\cdots,x_{ik_i})^T$, k_i's are suitable integers satisfying $\sum_{i=1}^{p} k_i = n$. Without loss of generality, suppose $1 \le k_1 \le k_2 \le \cdots \le k_p \le n$.

In [3], Gauthier and Bornard illustrated that under a uniform observability condition, the autonomous system

$$\begin{aligned}\dot{z} &= f(z)\\ y &= h(z)\end{aligned} \tag{1.2}$$

is transformed into the canonical form (1.1) by the following change of coordinates

$$\begin{aligned}x &= \Phi(z)\\ &= (h_1(z),\cdots,L_f^{k_1}h_1(z);\cdots;h_p(z),\cdots,L_f^{k_p}h_p(z)))^T\end{aligned}$$

where $z\in\mathbb{R}^n$ and $y\in\mathbb{R}^p$ are the system state and output, respectively. The vector fields $f:\mathbb{R}^n\to\mathbb{R}^n$ and $h:\mathbb{R}^n\to\mathbb{R}^p$ are smooth, with $n\ge p\ge 1$.

For the autonomous system (1.2), a common approach for the observer design is to find a change of coordinates and an output injection so that (1.2) can be transformed into the so-called observer form. The approach was first introduced by Krener and Isidori [10] and Bestle and Zeitz [1], in the single-output case (i.e., $p=1$), and then was generalized to the multi-output case by Krener and Respondek [12], Xia and Gao [21] and to discrete-time nonlinear systems by Lin and Byrnes [18]. More recent extensions can be found in the papers by Kazantzis and Kravvaris [7], and Krener and Xiao [13].

The observer form based design method was further extended by Rudolph and Zeitz [19] to multi-output autonomous systems with a block triangular observer form, which essentially requires $f_i(x)$ in (1.1) to have certain triangular structure. In the work [20], an explicit form of nonlinear observer was presented by Shim et al. for a class of multi-output multi-input (MIMO) nonlinear systems in a block triangular form. However, it is required that the bounds of the control inputs and system states be known. The nonlinearities of the systems are assumed to be Lipschitz with a known Lipschitz constant. In the paper by Krener and Kang [11], a step-by-step, local observer design method was developed for MIMO nonlinear control systems which are also in a block-triangular form. An interesting feature of the paper [11] is that the observer gains are nonlinear functions of the estimated states and recursively designed.

In this work, we consider the observer design for the observable canonical form (1.1) which *does not have a block-triangular structure,* because the nonlinearities $f_i(\cdot)$'s in (1.1) depend on the entire system states and all the sub-blocks of system (1.1) are coupled each other. To remove the block-triangular structure restriction in the previous work, we make the following assumption in this paper.

Assumption 1.1: For every $x(0)=x_0\in\mathbb{R}^n$, the corresponding solution trajectory $x(x_0,t)$ of the observable system (1.1) uniquely exists and is globally bounded on

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$[0, +\infty)$. That is, there is an unknown constant $C \geq 0$ depending on the initial condition x_0, such that

$$|x_{ij}(x_0, t)| \leq C,\ i = 1, \cdots, p;\ j = 1, \cdots, k_i;\ \forall t \in [0, \infty).$$

Assumption 1.1 is a mild condition for autonomous systems (without control), because it covers an important class of dynamic systems such as the Van der Pol equation and Duffing oscillator [5], [13] — both of them are *unstable* at the origin but nevertheless have *globally bounded* solution trajectories from any initial condition. On the other hand, the boundedness condition excludes the class of nonlinear systems with unbounded solutions or having a finite escape time, and hence is somewhat restrictive. This is, however, a trade-off for removing the block-triangular structure assumption.

With the aid of Assumption 1.1, a universal-like global observer can be designed for the multi-output autonomous system (1.1). Following the spirit of our recent work [16], we propose, in section II, an adaptive observer scheme in which a delicate rescaling technique is employed to deal with the inter-coupling terms $f_i(x)'s$ in (1.1) that consist of the entire system states. Due to the lack of the bound information of the solution trajectories, a saturation technique [8] is used in the construction of multivariable observers but the saturation threshold is tuned by a universal control law instead of being a prescribed constant. As done in the single-output case, the observer gain needs to be tuned adaptively. As a result, the proposed observer is a dynamic system with dimension of $n + 2$.

In addition to the main result presented in section II, we present in section III an extension of the global observer design scheme for a class of detectable systems. In section IV, the problem of global observer design is discussed for a class of systems with control inputs. To illustrate the validity of the results, two examples are given in section V. Conclusions are drawn in section VI.

Due to the space limitation, the proofs of the main results in the paper are omitted.

II. Dynamic High-Gain Observers for Observable systems

In this section, we will propose a constructive observer design scheme for the globally observable system (1.2) which satisfies the Assumption 1.1.

To introduce the main result, we first recall the definition of a unit saturation function.

Definition 2.1: A unit saturation function $\text{sat}(s)$ is defined as

$$\text{sat}(s) = \begin{cases} 1 & \text{if } s > 1 \\ s & \text{if } |s| \leq 1 \\ -1 & \text{if } s < -1 \end{cases} \tag{2.1}$$

From the definition, it is not difficult to show that

Lemma 2.2: Given real numbers s_1, s_2 and $m > 0$, suppose that $|s_1| \leq m$. Then,

$$|s_1 - m\text{sat}(\frac{s_2}{m})| \leq |s_1 - s_2|. \tag{2.2}$$

For any $x \in \mathbb{R}^n$ and $m > 0$, define mapping $\text{sat}_m : \mathbb{R}^n \to [-\mathrm{m}, \mathrm{m}]^n$ as

$$\text{sat}_m(x) := \big(m\text{sat}(\frac{x_1}{m}), m\text{sat}(\frac{x_2}{m}), \cdots, m\text{sat}(\frac{x_n}{m})\big)$$

Now, we are ready to state the main theorem of the paper.

Theorem 2.3: For the multi-output system in observer canonical form (1.1), suppose the Assumption 1.1 holds. Then, there exists a global observer. In particular, a globally convergent observer can be constructed as

$$\begin{aligned}
\dot{\hat{x}}_{i1} &= \hat{x}_{i2} + (MN)a_{i1}(y_i - \hat{x}_{i1}) \\
\dot{\hat{x}}_{i2} &= \hat{x}_{i3} + (MN)^2 a_{i2}(y_i - \hat{x}_{i1}) \\
&\vdots \\
\dot{\hat{x}}_{i,k_i-1} &= \hat{x}_{i,k_i} + (MN)^{k_i-1} a_{i,k_i-1}(y_i - \hat{x}_{i1}) \\
\dot{\hat{x}}_{i,k_i} &= f_i\big(\text{sat}_N(\hat{x})\big) + (MN)^{k_i} a_{i,k_i}(y_i - \hat{x}_{i1}) \\
\dot{N} &= \gamma \sum_{i=1}^{p} \Big(\frac{y_i - \hat{x}_{i1}}{(MN)^{k_p - k_i + 1}}\Big)^2, \quad N(0) = 1 \\
\dot{M} &= -M + \Delta(N), \qquad M(0) = 1
\end{aligned} \tag{2.3}$$

where $a_{ij} > 0, i = 1, \cdots, p, j = 1, \cdots, k_i$ are the coefficients of the Hurwitz polynomials $p_i(s) = s^{k_i} + \sum_{j=1}^{k_i} a_{ij} s^{k_i - j}$, $\gamma \geq 1$ is a prescribed constant, and $\Delta(N) \geq 1$ is a smooth function which can be determined explicitly.

Moreover, all the states of the closed-loop system (1.1)-(2.3) are well-defined and bounded on $[0, \infty)$, and,

$$\lim_{t\to\infty} [x(x_0, t) - \hat{x}(\hat{x}_0, t)] = 0, \quad \forall (x_0, \hat{x}_0) \in \mathbb{R}^n \times \mathbb{R}^n.$$

Remark 2.4: (2.3) is a universal-like high-gain observer that is motivated by the works [22], [6] and [15]. Different from the traditional high-gain observer [4] [9], the observer gain of (2.3) is composed of two parts. One is the moving saturation level N(t) which needs to be tuned in a manner similar to the one in [15], [16]. The other one is $M(t)$, which is used to recover the offset of $f_i(sat_N(\hat{x}))$ from $f_i(x)$, to be updated through a linear ODE driven by a nonlinear function of $N(t)$. The introduction of non-constant gains $N(t)$ and $M(t)$ enables us to deal with issue of the *unknown bound* of the solution trajectories of the observable system (1.1) or (1.2).

It should be mentioned that $\Delta(N)$ in the observer (2.3) can be calculated directly based on the observable system (1.1), in particular, by the nonlinear functions $f_i(x)$'s. To make this point clear, we introduce the following technical lemma.

Lemma 2.5: (Refer to [17]) Let $g : \mathbb{R}^n \to \mathbb{R}$ be a C^1 real-valued function. Then, there exist two smooth functions $\alpha, \beta : [0, +\infty) \to [1, +\infty)$, such that $\forall x, z \in \mathbb{R}^n$,

$$|g(x) - g(z)| \leq \alpha(||x||)\beta(||z||) \sum_{i=1}^{n} |x_i - z_i|. \tag{2.4}$$

Using the inequality (2.4), $\big|f_i(x) - f_i(\text{sat}_N(\hat{x}))\big|$ can be estimated as follows. By Assumption 1, $||x(x_0, t)|| \leq C,\ \forall t \geq 0$. Since $||\text{sat}_N(\hat{x})|| \leq N$, by Lemma 2.5, for each

$i = 1, 2, \cdots, p$, there exist two smooth positive functions $\alpha_i(\cdot)$ and $\beta_i(\cdot)$ such that

$$\begin{aligned} &\big|f_i(x) - f_i(\mathsf{sat}_N(\hat{x}))\big| \\ &\leq \alpha_i(C)\beta_i(N)\sum_{i=1}^{p}\sum_{j=1}^{k_i}\big|x_{ij} - N\mathsf{sat}(\frac{\hat{x}_{ij}}{N})\big| \end{aligned} \quad (2.5)$$

Denote $\alpha(C) = \sum_{i=1}^{p}\alpha_i(C), \beta(N) = \sum_{i=1}^{p}\beta_i(N)$, then one can simply choose

$$\Delta(N) = \beta^2(N) \geq 1. \quad (2.6)$$

In the next subsection, it will be shown that such a choice of $\Delta(N)$ suffices to ensure the dynamic system (2.3) being a globally convergent observer of system (1.1).

To sum up, a global observer for the observable system (1.1) with bounded solution trajectories can be constructed in three steps:

Step 1. Pick a suitable $\gamma > 0$ and choose constants $a_{ij} > 0$, $i = 1, \cdots, p, j = 1, 2, \cdots, k_i$, such that $p_i(s) = s^{k_i} + \sum_{j=1}^{k_i-1} a_{ij}s^{k_i-j}$ is Hurwitz;

Step 2. Use inequality (2.5) to estimate $\big|f_i(x) - f_i(\mathsf{sat}_N(\hat{x}))\big|$ and find $\beta(N) \geq 1$. Then, compute $\Delta(N) = \beta^2(N)$;

Step 3. With the obtained parameters γ, a_{ij}'s and $\Delta(N)$, design the observer (2.3).

Remark 2.6: It is worth pointing out that the dynamic update law of M can be modified as $\dot{M} = -\sigma M + \Delta(N), \sigma > 0, \Delta(N) \geq \sigma$ without affecting the argument in the above proof. A bigger σ makes the convergence of M faster and the gain $L = MN$ smaller, however, the convergence of the estimation slower.

Using Theorem 2.3, it is easy to obtain a corollary which is devoted to the design of a global observer for observable systems in a lower-triangular form:

$$\begin{aligned} \dot{z}_{i1} &= z_{i2} + f_{i1}(z_1) \\ \dot{z}_{i2} &= z_{i3} + f_{i2}(z_1, z_2) \\ &\vdots \\ \dot{z}_{i,k_i-1} &= z_{i,k_i} + f_{i,k_i-1}(z_1, z_2, \cdots, z_{k_i-1}) \\ \dot{z}_{i,k_i} &= f_{i,k_i}(z) \\ y &= z_1 \end{aligned} \quad (2.7)$$

where $1 < k_1 \leq \cdots \leq k_p$ and $\sum_{i=1}^{p}k_i = n$, $z_i = (z_{1i}, z_{2i}, \cdots, z_{p,i})^T$, if $1 \leq i \leq k_1$; $z_i = (z_{li}, z_{2i}, \cdots, z_{p,i})^T$, if $k_l < i \leq k_p, l = 1, \cdots, p-1$; $z = (z_1 \cdots, z_{k_p})$ are states and $y = z_1 = (z_{11}, z_{21}, \cdots, z_{p1})^T \in \mathbb{R}^{\mathrm{p}}$ are the outputs. $f_{ij}(\cdot), i = 1, \cdots, p, j = 1, \cdots, k_i$ are smooth functions with $f_{ij}(0, \cdots, 0) = 0$.

Due to the lower-triangular structure, one can explicitly construct a global change of coordinates $x = \Psi(z)$ which renders system (2.7) globally diffeomorphic to system (1.1). As a consequence, we have the following conclusion.

Corollary 2.7: Assume that all the solution trajectories of the lower-triangular system (2.7) from any initial condition are well-defined and bounded on $[0, +\infty)$. Then, a globally convergent observer exists and can be explicitly constructed.

III. Global Observer Design for Detectable Systems

This section is devoted to the design of global observers for a class of detectable nonlinear systems. Consider a class of autonomous systems of the form

$$\begin{aligned} \dot{\eta} &= A_u\eta + \Psi(y) \\ \dot{x}_{i1} &= x_{i2} \\ &\vdots \\ \dot{x}_{i,k_i-1} &= x_{i,k_i} \\ \dot{x}_{i,k_i} &= f_i(x), \qquad i = 1, 2, \cdots, p \\ y &= (y_1, y_2, \cdots, y_p)^T = (x_{11}, x_{21}, \cdots, x_{p1})^T \end{aligned} \quad (3.1)$$

where $\eta \in \mathbb{R}^{\mathrm{n-r}}$ and $x \in \mathbb{R}^{\mathrm{r}}$ are the system states, $y \in \mathbb{R}^{\mathrm{p}}$ are the outputs, and $1 < k_1 \leq k_2 \leq \cdots \leq k_p$, $\sum_{i=1}^{p}k_i = r$, $\Psi(y)$ is a continuous function and $f_i(\cdot)$'s are a smooth functions vanishing at origin.

Clearly, the state $\eta \in \mathbb{R}^{\mathrm{n-r}}$ is unobservable from the output y. This is because η has no influence on the system output. However, if the matrix A_u is Hurwitz, one can still design a global observer for the autonomous system (3.1) under the condition that the x-subsystem is bounded.

Theorem 3.1: Suppose the x-subsystem of (3.1) satisfies the bounded assumption in the sense of Assumption 1.1, and A_u is a Hurwitz matrix. Then, a global observer can be constructed for the system (3.1) in the following way:

$$\begin{aligned} \dot{\hat{\eta}} &= A_u\hat{\eta} + \Psi(y) \\ \dot{\hat{x}}_{i1} &= \hat{x}_{i2} + (MN)a_{i1}(y_i - \hat{x}_{i1}) \\ &\vdots \\ \dot{\hat{x}}_{i,k_i-1} &= \hat{x}_{i,k_i} + (MN)^{k_i-1}a_{i,k_i-1}(y_i - \hat{x}_{i1}) \\ \dot{\hat{x}}_{i,k_i} &= f_i\big(\mathsf{sat}_N(\hat{x})\big) + (MN)^{k_i}a_{i,k_i}(y_i - \hat{x}_{i1}) \\ \dot{N} &= \gamma\sum_{i=1}^{p}\Big(\frac{y_i - \hat{x}_{i1}}{(MN)^{k_p-k_i+1}}\Big)^2, \qquad N(0) = 1 \\ \dot{M} &= -M + \Delta(N), \qquad M(0) = 1 \end{aligned} \quad (3.2)$$

where $a_{ij} > 0, i = 1, \cdots, p, j = 1, \cdots, k_i$ are the coefficients of the Hurwitz polynomials $h_i(s) = s^{k_i} + \sum_{j=1}^{k_i} a_{ij}s^{k_i-j}$, $\gamma \geq 1$ is a prescribed constant.

The observer (3.2) guarantees that all the states of the closed-loop system (3.1)-(3.2) are well-defined and bounded on $[0, \infty)$. In addition, $\lim_{t\to\infty}[\eta(\eta_0, t) - \hat{\eta}(\eta_0, t)] = 0$, $\lim_{t\to\infty}[x(x_0, t) - \hat{x}(\hat{x}_0, t)] = 0$, $\forall(\eta_0, x_0) \in R^n, (\hat{\eta}_0, \hat{x}_0) \in \mathbb{R}^{\mathrm{n}}$.

Remark 3.2: Theorem 3.1 suggests that, in terms of observer *design*, the observability is not a necessary condition. This is similar to the linear case, i.e., an unobservable yet detectable system still permits the existence of an observer.

Remark 3.3: Theorem 3.1 remains true if the unobservable sub-system is replaced by

$$\dot{\eta} = \varphi(y)(A_u\hat{\eta} + \Psi(y)), \quad \varphi(y) > 0. \quad (3.3)$$

In this case, one can still design a global observer using a manner similar to the one suggested in Theorem 3.1, with a slight modification.

IV. Observers for a class of Nonlinear Systems with Control Inputs

We now discuss briefly the observer design problem for the following multi-output multi-input (MIMO) nonlinear system

$$\begin{aligned}\dot{x}_{i1} &= x_{i2} + g_{i1}(y,u)\\ \dot{x}_{i2} &= x_{i3} + g_{i2}(x,u)\\ &\vdots\\ \dot{x}_{i,k_i-1} &= x_{i,k_i} + g_{i,k_i-1}(x,u)\\ \dot{x}_{i,k_i} &= f_i(x) + g_{i,k_i}(x,u)\\ y &= (x_{11}, x_{21}, \cdots, x_{p1})^T \end{aligned} \tag{4.1}$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{i,k_i})^T$, $x = (x_1, \cdots, x_p)^T \in \mathbb{R}^{\mathrm{n}}$, $u \in \mathbb{R}^{\mathrm{m}}$ and $y \in \mathbb{R}^{\mathrm{p}}$ are the system state, input and output, respectively, $k_1 \le k_2 \le \cdots \le k_p$, $\sum_{i=1}^{p} k_i = n$. The functions $g_{i,j}(\cdot)$ and $f_i(\cdot)$ are smooth with $g_{i,j}(0,0) = 0$ and $f_i(0) = 0$.

We assume that the function $g_{ij}(x,u) := g_{ij}(y, \underline{x}, u)$, with $x = (y^T,\ \underline{x}^T)^T$ and $\underline{x} = \mathrm{col}(x_{ij}, i = 1, \cdots, p; j = 2, \cdots, k_i) \in R^{n-p}$, satisfies the following condition.

Assumption 4.1: For $i = 1, \cdots, p$ and $j = 2, \cdots, k_i$,

$$|g_{ij}(y, \underline{x}, u) - g_{ij}(y, \underline{\hat{x}}, u)| \le c(x,u)\big(\sum_{s=1}^{i}\sum_{l=2}^{j} |x_{sl} - \hat{x}_{sl}|\big)$$

where $\underline{\hat{x}} = \mathrm{col}(\hat{x}_{ij}, i = 1, \cdots, p; j = 2, \cdots, k_i) \in \mathbb{R}^{\mathrm{n-p}}$, and $c(\cdot,\cdot) \ge 0$ is a smooth function.

Assumption 4.2: For any control input $u(t)$ in the compact set $U \subset \mathbb{R}^{\mathrm{m}}$ and any initial condition $x_0 \in \mathbb{R}^{\mathrm{n}}$, the corresponding solution trajectory $x_u(x_0,t)$ of the controlled system (4.1) is well-defined over the interval $[0, +\infty)$ and $x_u(x_0,t)$ is globally bounded, i.e.

$$||x_u(x_0,t)|| \le C.$$

- In [20], a nonlinear observer was presented for a class of MIMO nonlinear systems. The system studied there is of a block triangular form. Moreover, it is required that the bounds of the system input and state be known. The system nonlinearities are assumed to satisfy a Lipschitz condition with a known Lipschitz constant.
- In [11], a step-by-step local observer design method was proposed for a class of multi-input multi-output nonlinear control systems. The systems under consideration are also in a block-triangular form, and the observer gains are nonlinear functions of the estimated states. Due to the local design feature, the boundedness condition is automatically satisfied.
- Assumption 4.2 basically requires that all the solution trajectories do not blow up under bounded control. It contains, for instance, bounded-input/bounded-state (BIBS) systems. It should be noticed that a key feature of the proposed observer does not need the bound information of the solution trajectories.

Under the two assumptions above, we can design a global observer for the MIMO system (4.1) by following the spirit of observer design method in section II.

Theorem 4.3: For the MIMO nonlinear control system (4.1), suppose Assumptions 4.1 and 4.2 hold. Then, a global observer can be designed for the controlled systems (4.1) as

$$\begin{aligned}\dot{\hat{x}}_{i1} &= \hat{x}_{i2} + (MN)a_{i1}(y_i - \hat{x}_{i1}) + g_{i1}(y,u)\\ \dot{\hat{x}}_{i2} &= \hat{x}_{i3} + (MN)^2 a_{i2}(y_i - \hat{x}_{i1}) + g_{i2}(y, \underline{\hat{x}}, u)\\ &\vdots\\ \dot{\hat{x}}_{i,k_i} &= f_i\big(\mathrm{sat}_N(\hat{x})\big) + (MN)^{k_i} a_{i,k_i}(y_i - \hat{x}_{i1})\\ &\quad + g_n(y, \underline{\hat{x}}, u)\\ \dot{N} &= \gamma \sum_{i=1}^{p}\Big(\frac{y_i - \hat{x}_{i1}}{(MN)^{k_p - k_i + 1}}\Big)^2, \qquad N(0) = 1\\ \dot{M} &= -M + \Delta(N), \qquad\qquad M(0) = 1 \end{aligned} \tag{4.2}$$

where $a_{ij} > 0, i = 1, \cdots, p, j = 1, \cdots, k_i$ are the coefficients of the Hurwitz polynomials $s^{k_i} + \sum_{j=1}^{k_i} a_{ij} s^{k_i - j}$, $\gamma \ge 1$ is a prescribed constant, and $\Delta(N) \ge 1$ is a smooth function which can be determined explicitly. Moreover, all the states of the closed-loop system (4.1)-(4.2) are well-defined and bounded on $[0, \infty)$. In addition,

$$\lim_{t\to\infty}[x(x_0,t) - \hat{x}(\hat{x}_0,t)] = 0, \quad \forall (x_0, \hat{x}_0) \in \mathbb{R}^{\mathrm{n}} \times \mathbb{R}^{\mathrm{n}}.$$

The proof of this theorem can be carried out by modifying suitably the argument of Theorem 2.3. The boundedness property of x and u has to be used, but the bound can be unknown.

V. Examples and Simulations

In this section, we give two examples to illustrate the applications of the observer design methods proposed in this paper.

Example 5.1: Consider the two-output observable autonomous systems

$$\begin{aligned}\dot{x}_{11} &= x_{12}\\ \dot{x}_{12} &= -x_{11} - x_{12}^3 + x_{23} + x_{21}^3\\ \dot{x}_{21} &= x_{22}\\ \dot{x}_{22} &= x_{23}\\ \dot{x}_{23} &= -3x_{21}^2 x_{22} - x_{22} - x_{12}\\ y &= (y_1, y_2)^T = (x_{11}, x_{21})^T \end{aligned} \tag{5.1}$$

This system is of the form (1.1). Choosing Lyapunov function $V(x) = \frac{1}{2}[x_{11}^2 + x_{12}^2 + \frac{1}{2}x_{21}^4 + x_{22}^2 + (x_{21}^3 + x_{23})^2]$, one can see that the derivative of $V(x)$ along the trajectories of (5.1) satisfies $\dot{V} = -x_{12}^4 \le 0$, which implies that the system is stable but not asymptotically stable. Hence, all the solutions trajectories of (5.1) are globally bounded, and the design method proposed in Theorem 2.3 can be applied.

To find the function $\Delta(N)$, we first compute $\beta_1(N)$ and $\beta_2(N)$ from $f_1(x) = -x_{11} - x_{12}^3 + x_{23} + x_{21}^3$ and $f_2(x) =$

$-3x_{21}^2x_{22} - x_{22} - x_{12}$. By the mean value theorem, there is a $\xi \in \mathbb{R}^5$ between x and $\mathrm{sat}_N(\hat{x})$, such that

$$
\begin{aligned}
& |f_1(x) - f_1(\mathrm{sat}_N(\hat{x}))| \\
= \; & |\sum_{i=1}^{2} \frac{\partial f_1}{\partial \xi_{1i}}(\xi)(x_{1i} - N\mathrm{sat}(\frac{\hat{x}_{1i}}{N})) \\
& + \sum_{i=1}^{3} \frac{\partial f_1}{\partial \xi_{2i}}(\xi)(x_{2i} - N\mathrm{sat}(\frac{\hat{x}_{2i}}{N})| \\
\leq \; & (2 + 3\xi_{12}^2 + 3\xi_{21}^2) \cdot \big(\sum_{i=1}^{2} |x_{1i} - N\mathrm{sat}(\frac{\hat{x}_{1i}}{N})| \\
& + \sum_{i=1}^{3} |x_{2i} - N\mathrm{sat}(\frac{\hat{x}_{2i}}{N})| \big) \\
\leq \; & (4 + 6(C+N)^2) \cdot \big(\sum_{i=1}^{2} |x_{1i} - N\mathrm{sat}(\frac{\hat{x}_{1i}}{N})| \\
& + \sum_{i=1}^{3} |x_{2i} - N\mathrm{sat}(\frac{\hat{x}_{2i}}{N})| \big) \\
\leq \; & 6(C^2+1)(N^2+1)\big(\sum_{i=1}^{2} |x_{1i} - N\mathrm{sat}(\frac{\hat{x}_{1i}}{N})| \\
& + \sum_{i=1}^{3} |x_{2i} - N\mathrm{sat}(\frac{\hat{x}_{2i}}{N})| \big).
\end{aligned}
$$

Thus, $\beta_1(N) = N^2 + 1$.

Similarly, it is deduced from $f_2(x)$ that $\beta_2(N) = N^2+1$. Hence, $\beta(N) = \beta_1(N) + \beta_2(N) = 2N^2 + 2$ and $\Delta(N) = \beta^2(N) = 4(N^2+1)^2$. Choose $a_{11} = a_{12} = a_{21} = a_{23} = 1, a_{22} = 3$, $\gamma = 8$. Then, the observer for the autonomous system (5.1) can be designed as

$$
\begin{aligned}
\dot{\hat{x}}_{11} &= \hat{x}_{12} + MN(x_{11} - \hat{x}_{11}) \\
\dot{\hat{x}}_{12} &= -N\mathrm{sat}(\frac{\hat{x}_{11}}{N}) - N^3\mathrm{sat}^3(\frac{\hat{x}_{12}}{N}) + N\mathrm{sat}(\frac{\hat{x}_{23}}{N}) \\
& \quad + N^3\mathrm{sat}^3(\frac{\hat{x}_{21}}{N}) + M^2N^2(x_{11} - \hat{x}_{11}) \\
\dot{\hat{x}}_{21} &= \hat{x}_{22} + MN(x_{21} - \hat{x}_{21}) \\
\dot{\hat{x}}_{22} &= \hat{x}_{23} + 3M^2N^2(x_{21} - \hat{x}_{21}) \\
\dot{\hat{x}}_{23} &= -3N^3\mathrm{sat}^2(\frac{\hat{x}_{21}}{N})\mathrm{sat}(\frac{\hat{x}_{22}}{N}) - N\mathrm{sat}(\frac{\hat{x}_{21}}{N}) \\
& \quad - N\mathrm{sat}(\frac{\hat{x}_{12}}{N}) + M^3N^3(x_{21} - \hat{x}_{21}) \\
\dot{N} &= \frac{8}{M^4N^4}\big((x_{11} - \hat{x}_{11})^2 + M^2N^2(x_{21} - \hat{x}_{21})^2\big) \\
\dot{M} &= -M + 4(N^2+1)^2 \qquad (5.2) \\
& \quad N(0) = 1, \qquad M(0) = 1
\end{aligned}
$$

Fig. 1 illustrates the transient response of the observer (5.2) and the system (5.1) starting from the initial conditions $(x_{11}^0, x_{12}^0, x_{21}^0, x_{22}^0, x_{23}^0, \hat{x}_{11}^0, \hat{x}_{12}^0, \hat{x}_{21}^0, \hat{x}_{22}^0, \hat{x}_{23}^0) = (2, 2, 2, -2, 4, 3, 1, 1, -2, 2)$.

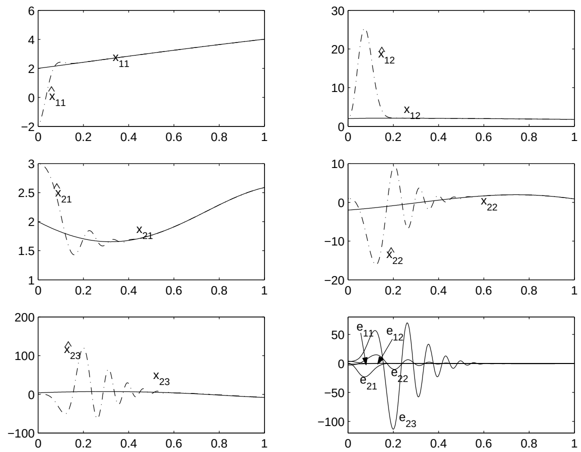

Fig. 1. Observation of a 5-dimension 2-output system

Example 5.2: Consider the observation of a point-mass satellite model (see, for instance, [2]):

$$
\begin{aligned}
\dot{\rho} &= v \\
\dot{v} &= \rho\omega^2 - \theta_1 \frac{1}{\rho^2} + \theta_2 u_1 \\
\dot{\phi} &= \omega \qquad (5.3) \\
\dot{\omega} &= -\frac{1}{\rho}(2v\omega + \theta_2 u_2)
\end{aligned}
$$

in which (ρ, ϕ) denotes the position of the satellite in polar coordinates on the plane, v is the radial velocity, ω is the angular velocity and u_1, u_2 are the radial and tangential thrust, respectively. We assume that the measurable signals are

$$y_1 = \rho, \quad y_2 = \phi.$$

Consider the case when the parameters $\theta_1 = 4$ and $\theta_2 = 1$, while the control inputs $u_1 = 4/\rho^2 - \rho - v$ and $u_2 = \frac{\phi}{\rho}$. Then, it is easy to verify that the system states are globally bounded, by using the Lyapunov function $V = \frac{1}{2}(\rho^2 + v^2 + \phi^2 + \rho^2\omega^2)$ whose derivative is $\dot{V} = -v^2 \leq 0$. The state trajectories of the closed-loop system are shown in Fig. 2.

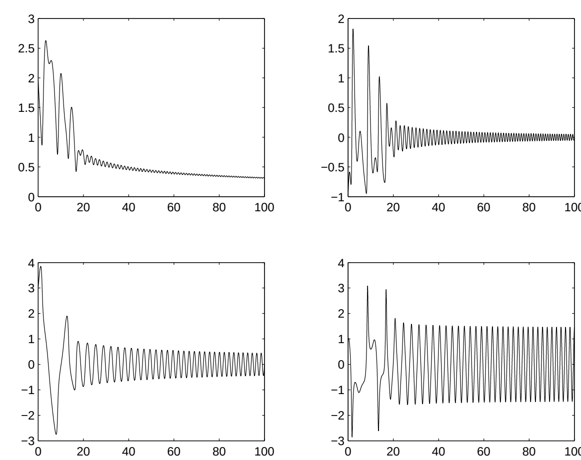

Fig. 2. State trajectories of point-mass satellite model

For the closed-loop system, we can design a global observer of the form

$$
\begin{aligned}
\dot{\hat{\rho}} &= \hat{v} + 6MN(\rho - \hat{\rho}), \quad \hat{\rho}(0) > 0 \\
\dot{\hat{v}} &= N^3 \mathrm{sat}(\frac{\hat{\rho}}{N}) \mathrm{sat}^2(\frac{\hat{\omega}}{N}) - N \mathrm{sat}(\frac{\hat{\rho}}{N}) - N \mathrm{sat}(\frac{\hat{v}}{N}) \\
&\quad + 9(MN)^2(\rho - \hat{\rho}) \\
\dot{\hat{\phi}} &= \hat{\omega} + MN(\phi - \hat{\phi}) \\
\dot{\hat{\omega}} &= -\frac{1}{N \mathrm{sat}(\frac{\hat{\rho}}{N})} \left(2N^2 \mathrm{sat}(\frac{\hat{v}}{N}) \mathrm{sat}(\frac{\hat{\omega}}{N}) + \frac{\mathrm{sat}(\frac{\hat{\phi}}{N})}{\mathrm{sat}(\frac{\hat{\rho}}{N})} \right) \\
&\quad + (MN)^2(\phi - \hat{\phi}) \\
\dot{N} &= \frac{5}{(MN)^2} \Big((\rho - \hat{\rho})^2 + (\phi - \hat{\phi})^2 \Big), \qquad N(0) = 1 \\
\dot{M} &= -M + (N^2 + 1)^2, \qquad M(0) = 1 \qquad (5.4)
\end{aligned}
$$

Figure 3 illustrates the simulation results of the closed-loop system and the observer (5.4) starting from the initial conditions $(\rho^0, v^0, \phi^0, \omega^0, \hat{\rho}^0, \hat{v}^0, \hat{\phi}^0, \hat{\omega}^0) = (2, -1, 3, 1, 4, 2, 1, 2)$

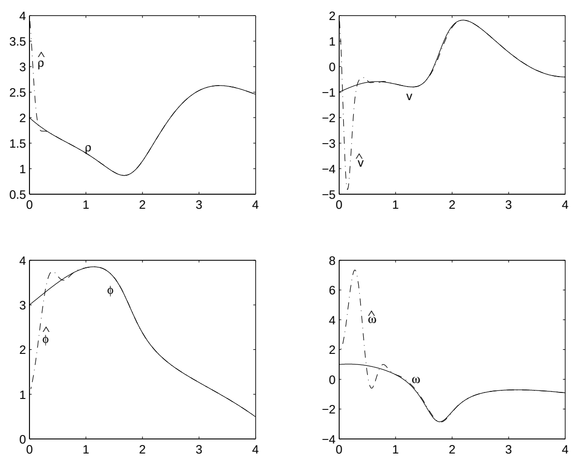

Fig. 3. Observation of point-mass satellite model

VI. Conclusions

Under the global boundedness and observability conditions, we have shown that a globally convergent observer can be explicitly designed for the multi-output autonomous system (1.1) or (1.2) without requiring a block-triangular structure nor imposing restrictions on the coupling relations between each sub-block. The constructed observer is of high-gain type but different from the traditional one [9] in the sense that the observer gains here are composed of two time-varying components $M(t)$ and $N(t)$, both of them must be adaptively updated in order to deal with the issue of the unknown bound of the solution trajectories. The gain update law is reminiscent from the recent work [15] on universal output feedback control of nonlinear systems with unknown parameters. It was also showed that the proposed observer design technique can be extended to a class of detectable systems and multi-input/multi-output (MIMO) nonlinear systems with bounded solution trajectories, such as bounded-input/bounded-state (BIBS) systems.

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